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Quantum Methods in Field Theories with Singular Higher Derivative Lagrangians

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The path integral quantization for higher derivative Chern–Simons theories in $(2+1)$ dimensions coupled to fermions is treated. The diagrammatic and the Feynman rules are constructed and the regularization and renormalization of this higher derivative model are analyzed in the framework of the perturbation theory. Finally, the unitarity problem related to the possible appearance of ghost states with negative norm is also discussed.

1. INTRODUCTION

Quantum field theories in $(2+1)$ dimensions have been studied with increasing interest in recent years because many interesting problems are present in the $(2+1)$-dimensional physics (Deser *et al.*, 1992*a,b*, 1988; Dunne *et al.*, 1989; Jackiw and Templeton, 1981; Matsuyama, 1990*a,b*; Li and Ni, 1990; Avdeev *et al.*, 1992; Odintsov, 1992; Plyushchay, 1992; Jackiw and Weinberg, 1990; Hlousek and Spector, 1990; Kogan, 1991; Chon *et al.*, 1993). On the other hand, dynamical systems described by means of singular higher derivative Lagrangians have also been investigated by several authors and is a problem of current research in quantum field theory (Ellis, 1975; Leon and Rodriguez, 1985; Kerstyen, 1988; Nesterenko, 1989; Li, 1991). These kinds of quantum theories containing higher derivative terms in the action are also frequently used in topics of condensed matter, such as high-T_c superconductivity.

In a recent paper (Greco *et al.*, 1994) we considered the canonical and the path integral quantum formalisms for constrained Hamiltonian system

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with a singular higher-order Lagrangian which describes the Chern–Simons (CS) theories in $(2+1)$ dimensions coupled to fermionic matter. Perhaps one of the reasons these theories have not been treated intensively in spite of their physical interest is due to the appearance of ghost states with negative norm, which can cause unitarity violation (Hawking, 1987). However, they have also some kind of attraction. When higher derivative terms are added to the Lagrangian, the convergence of the corresponding Feynman diagrams can be improved (Nesterenko, 1989; Alvarez-Gaumé *et al.*, 1990).

Due to the presence of the volume form $\varepsilon^{\mu\nu\rho}$ in the $(2+1)$ CS theories, dimensional regularization cannot be used. Consequently, another mixed regularization method involving higher covariant derivative and the Pauli–Villars method is applied (Alvarez-Gaumé *et al.*, 1990). This procedure preserves also the gauge invariance of the theory. Alvarez-Gaumé *et al.* (1990) showed how the technique to introduce higher derivative terms in the action improves the behavior of propagators at large momentum, rendering the theory less divergent.

It is necessary to emphasize that the perturbative formalism developed in Alvarez-Gaumé *et al.* (1990) is not a proper formalism for a higher derivative field theory. Really, in a higher derivative field theory the phase space is generated by means of the Ostrogradski transformation (Nesterenko, 1989; Li, 1991; Greco *et al.*, 1994). In fact, in Alvarez-Gaumé *et al.* (1990) higher derivative terms are added to the action with the unique purpose to render the theory regularized. At the end of the procedure, the multiplicative parameter Λ in front of the higher derivative terms, which has dimensions of mass and acts as a cutoff, is removed by taking the $\Lambda \to \infty$ limit.

The main motivation of the present paper arises from the point of view of the field theory itself. More precisely, we consider how to treat quantum field theories described by singular higher derivative Lagrangians. In particular, in this kind of theory there are two important problems that we want to consider. First, the possibility of unitarity violation. This naturally leads us to explore, in the framework of the path integral quantization, the states with negative norm related to the ghost state problem. Second, starting from a suitable definition of propagators and vertices and after a diagrammatic for the higher derivative model is constructed, we must study the number of divergent diagrams and give prescriptions about regularization and renormalization. The results must be confronted with those obtained for the corresponding model without higher derivative terms in the Lagrangian. Consequently, at least for this case, a response concerning the convenience or not of adding higher derivative terms in the Lagrangian density can be given.

The paper is organized as follows. In Section 2, we briefly recall the main results about the Lagrangian and Hamiltonian formalisms for systems with singular Lagrangians containing higher derivative terms. We consider the phase space definition and how we can go over from the Lagrangian description to the Hamiltonian description. This is necessary both to make possible the generalization of the Dirac's conjecture (Dirac, 1964) and to extend the Faddeev–Senjanovic path integral quantization method (Faddeev, 1970; Senjanovic, 1976) to these kinds of constrained Hamiltonian systems. In Section 3, we consider a particular system described by a singular second-order Lagrangian for the fermion coupling to CS theories in $(2+1)$ dimensions. The set of constraints is analyzed and the total Hamiltonian as a first-class dynamical quantity is given. In Section 4, by extending the Faddeev–Senjanovic method to higher derivative theory, we perform the path integral quantization. By defining an effective Lagrangian density, we study the Feynman rules and the diagrammatic corresponding to this coupled system. In Section 5, by means of a suitable definition of a "new" bosonic propagator, we analyze the one-loop structure of the theory from a perturbative point of view. Moreover, prescriptions about the ultraviolet behavior of the correction to the boson line, the correction to the fermion line, and the vertex correction can be done. Finally, in Section 6, we discuss the unitarity problem by analyzing the properties of the bosonic propagator we have defined.

2. PRELIMINARIES

We start by considering a second-order Lagrangian density of the form

$$\mathscr{L} = \mathscr{L}(A_i, \partial_\mu A_i, \partial_\mu \partial_\nu A_i) \tag{2.1}$$

for a set of fields A_i $(i = 1, 2, \ldots, n)$ and where $\mu, \nu = 0, 1, 2$ are space-time indices.

The Euler–Lagrange equations obtained from the variational principle are given by

$$\frac{\partial \mathscr{L}}{\partial A_i} - \partial_\mu \frac{\partial \mathscr{L}}{\partial(\partial_\mu A_i)} + \partial_\mu \partial_\nu \frac{\partial \mathscr{L}}{\partial(\partial_\mu \partial_\nu A_i)} = 0 \tag{2.2}$$

When we have in hand a second-order Lagrangian density as written in (2.1), the canonical variables are introduced according to the Ostrogradski (1850) transformation as follows.

Let $A_i^{(1)} = A_i$ and $A_i^{(2)} = \dot{A}_i$ be the dynamical field variables; consequently the canonical momenta are defined in the following way:

$$P_i^{(1)} = \frac{\partial \mathscr{L}}{\partial \dot{A}_i^{(1)}} - \partial_\mu \frac{\partial \mathscr{L}}{\partial(\partial_\mu A_i^{(2)})} \tag{2.3a}$$

$$P_i^{(2)} = \frac{\partial \mathscr{L}}{\partial \dot{A}_i^{(2)}} \tag{2.3b}$$

where we denote by $\dot{A}_i \equiv \partial_0 A_i$ the time derivative.

The Lagrangian density is called nonsingular (or nondegenerate) if the canonical conjugate variables $A_i^{(\alpha)}$ and $P_i^{(\alpha)}$ ($\alpha = 1, 2$) introduced above are all independent, i.e., they are not of the form

$$f^0(A_i^{(\alpha)}, P_i^{(\alpha)}) = 0 \tag{2.4}$$

In this case the extended Hessian matrix given by

$$H_{ij} = \frac{\partial^2 \mathscr{L}}{\partial \dot{A}_i^{(2)} \, \partial \dot{A}_j^{(2)}} \tag{2.5}$$

is nondegenerate ($\text{rank}\|H_{ij}\| = n$) and therefore the system of n Euler–Lagrange equations (2.2) is equivalent to a canonical system of $4n$ first-order equations,

$$\dot{A}_i^{(\alpha)} = \frac{\partial H}{\partial P_i^{(\alpha)}}, \qquad \dot{P}_i^{(\alpha)} = \frac{\partial H}{\partial A_i^{(\alpha)}} \tag{2.6}$$

The canonical Hamiltonian H is defined in the following way:

$$H = \dot{A}_i^{(\alpha)} P^{(\alpha)i} - \mathscr{L} = A_i^{(2)} P^{(1)i} + \dot{A}_i^{(2)} P^{(2)i} - \mathscr{L} \tag{2.7}$$

Otherwise, we say the Lagrangian density (2.1) is singular when a number $r \le n$ of constraints

$$f_a^0(A_i^{(\alpha)}, P_i^{(\alpha)}) = 0, \qquad a = 1, \dots, r \tag{2.8}$$

exists, in such a way that the extended Hessian matrix H_{ij} is degenerated ($\text{rank}\|H_{ij}\| = n - r$), and so equation (2.3b) is not soluble for all $\dot{A}_i^{(2)}$. The constraints (2.8) arising from equation (2.3b) are called primary constraints. By introducing a set λ^a of Lagrange multipliers it is possible to obtain the canonical equations for a constrained Hamiltonian system described by a singular second-order Lagrangian:

$$\dot{A}_i^{(\alpha)} = [A_i^{(\alpha)}, H_T], \qquad \dot{P}_i^{(\alpha)} = [P_i^{(\alpha)}, H_T] \tag{2.9}$$

where $[\cdot\,,\cdot]$ are the Poisson brackets and the extended Hamiltonian is given by

$$H_T = H_{\text{can}} + \lambda^a f_a^0 \tag{2.10}$$

By following this line, it is possible to obtain the generalized Noether theorems for a higher derivative constrained Hamiltonian system (Li, 1991). Moreover, as in the case of the dynamics for constrained Hamiltonian systems developed by Dirac (1964) in systems described by singular Lagrangians with higher derivative terms, by demanding the stationarity of the primary constraints, it is possible to give an algorithm analogous to the Dirac–Bergmann one. So, by writing

$$f_a^k = [f_a^{k-1}, H_T] \tag{2.11}$$

the algorithm must be continued until f_a^m satisfies

$$f_a^{m+1} = [f_a^m, H_T] = C_{ak}^b f_b^k \qquad (k \leq m) \tag{2.12}$$

All the constraints f_a^m are classified into two classes. A constraint f_a is called "first class" if $[f_a, f_b] = 0 \ (\text{mod}\, f_c)$ for all constraints; otherwise the constraints are "second class." Let us assume that we work with higher-order-derivative singular Lagrangians, for which there are no problems with Dirac's conjectures. Particular cases having objection or problems with the Dirac algorithm were treated and analyzed in the literature (Appleby, 1982; Castellani, 1982; Sugano and Kamo, 1982; Sugano and Kimura, 1983; Costa *et al.*, 1985).

On the other hand, it is well known that the gauge theories play an important role in the framework of modern field theories. These theories present local gauge invariances and each first-class constraint corresponds to a gauge invariance of the theory under local gauge transformation. There are simple cases in which all the constraints are first class, so it is not hard to construct the generator $G(A_i^{(\alpha)}, P_i^{(\alpha)})$ of the gauge transformation parametrized by an infinitesimal arbitrary function ε:

$$G = \sum_{k=0}^{m} \varepsilon^k G_k = \sum_{k=0}^{m} (D^k \varepsilon) G_k \tag{2.13}$$

Let us consider the variations

$$\delta A_i^{(\alpha)} = \sum_{k=0}^{m} \varepsilon^k \frac{\partial G_k}{\partial P_i^{(\alpha)}} \tag{2.14a}$$

$$\partial P_i^{(\alpha)} = -\sum_{k=0}^{m} \varepsilon^k \frac{\partial G_k}{\partial A_i^{(\alpha)}} \tag{2.14b}$$

When all the constraints G_k are first class satisfying the recursive relations

$$[G_0, H_T] = 0 \qquad (\text{mod primary constraints}) \tag{2.15a}$$

$$G_{k-1} + [G_k, H_T] = 0 \qquad (\text{mod primary constraints}) \tag{2.15b}$$

$$G_m = 0 \qquad (\text{mod primary constraints}) \tag{2.15c}$$

the generator G given in (2.13) is conservative, i.e., $\dot{G} = \partial G/\partial t + [G, H_T] = 0$ (mod primary constraints).

The model we will consider in the next section is more complicated because it has constraints of both first class and second class. Therefore in the framework of the path integral quantization, we must extend to higher derivative systems the Faddeev–Senjanovich method, valid when first- and second-class constraints are present.

3. HIGHER DERIVATIVE CS THEORIES

In Greco *et al.* (1994) we considered the following singular Lagrangian density:

$$\mathscr{L} = \mathscr{L}_{\text{top}} + \mathscr{L}_f + \mathscr{L}_{\text{int}} + \mathscr{L}_h \tag{3.1}$$

describing the fermionic matter couplet to higher derivative CS theories.

The different pieces in (3.1) are as follows:

(i) The electromagnetic Lagrangian density with a topological mass term, i.e., a CS term, is

$$\mathscr{L}_{\text{top}} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4\pi} \varepsilon^{\mu\nu\rho}\partial_\mu A_\nu A_\rho \tag{3.2a}$$

where the field strength tensor is written in terms of potentials in the usual way $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

(ii) The fermionic and interacting parts are, respectively,

$$\mathscr{L}_f = i\left(\frac{a+1}{2}\right)\bar{\psi}\gamma^\mu\partial_\mu\psi + i\left(\frac{a-1}{2}\right)\partial_\mu\bar{\psi}\gamma^\mu\psi - m\bar{\psi}\psi \tag{3.2b}$$

$$\mathscr{L}_{\text{int}} = e\bar{\psi}\gamma^\mu\psi A_\mu \tag{3.2c}$$

(iii) The part containing higher derivatives is given by

$$\mathscr{L}_h = -\frac{c}{4\pi}\partial_\rho F_{\mu\nu}\partial^\rho F^{\mu\nu} \tag{3.2d}$$

The constant κ in equation (3.2a) is the topological mass of the gauge field and its dimension is $(\text{length})^{-1}$; in equation (3.2d) the constant c has dimension $(\text{length})^1$. We will use the convention $\varepsilon^{012} = \varepsilon^{12} = 1$, the Minkowski metric $g_{\mu\nu}$ is $g_{\mu\nu} = \text{diag}(1, -1, -1)$, and the Dirac γ-matrices are $\gamma^0 = \sigma^3$, $\gamma^1 = i\sigma^1$, and $\gamma^2 = i\sigma^2$ (the σ's are the Pauli matrices).

After the Ostrogradski transformation is performed according to equations (2.3), the momenta $P^{(1)\mu}$, $P^{(2)\mu}$, $\bar{\Pi}^{(\alpha)}$, and $\Pi^{(\alpha)}$ canonically conjugate, respectively, to the independent field variables $A_\mu^{(1)} = A_\mu$, $A_\mu^{(2)} = \dot{A}_\mu$, ψ_α, and $\bar{\psi}_\alpha$ remain defined and are given by

$$P^{(1)0} = -\frac{c}{\pi}\,\partial_0\partial_i F^{0i} \tag{3.3a}$$

$$P^{(1)i} = F^{i0} + \frac{\kappa}{4\pi}\,\varepsilon^{ij}A_j + \frac{c}{\pi}\,(\nabla^2 F^{0i} + \partial_0\partial_j F^{ji}) - \partial_0 P^{(2)i} \tag{3.3b}$$

$$P^{(2)0} = 0 \tag{3.3c}$$

$$P^{(2)i} = -\frac{c}{\pi}\,\partial_0 F^{0i} \tag{3.3d}$$

$$\Pi_{(\alpha)}(x) = i\left(\frac{a-1}{2}\right)\gamma_0\psi_{(\alpha)} \tag{3.3e}$$

$$\bar{\Pi}_{(\alpha)}(x) = -i\left(\frac{a+1}{2}\right)\bar{\psi}_{(\alpha)}\gamma_0 \tag{3.3f}$$

where the Latin indices take the values $i, j = 1, 2$.

Going on with the Dirac algorithm, it is possible to find the Dirac brackets and to construct the canonical quantization (Greco *et al.*, 1994).

Summarizing the results, we can conclude that the Lagrangian density (3.1) describes a constrained Hamiltonian system which has three first-class constraints associated with the gauge symmetries of the system and two fermionic second-class constraints. The total Hamiltonian is given by

$$H_T = \int d^2x(\mathscr{H}_{\text{can}} + \beta^a\Phi_a) \tag{3.4}$$

where $\mathscr{H}_{\text{can}}$ is defined as

$$\mathscr{H}_{\text{can}} = A_\mu^{(2)}P^{(1)\mu} + \dot{A}_\mu^{(2)}P^{(2)\mu} + \dot{\bar{\psi}}_{(\alpha)}\Pi^{(\alpha)} + \bar{\Pi}^{(\alpha)}\dot{\psi}_{(\alpha)} - \mathscr{L} \tag{3.5}$$

The three first-class constraints Φ_a in equation (3.4) are given by

$$\Phi_1(x) = P^{(2)0}(x) \approx 0 \tag{3.6a}$$

$$\Phi_2(x) = -P^0(x) + \partial_i P^{(2)i}(x) \approx 0 \tag{3.6b}$$

$$\Phi_3(x) = -ie(\bar{\psi}_{(\alpha)}(x)\Pi^{(\alpha)}(x) + \bar{\Pi}^{(\alpha)}(x)\psi_{(\alpha)}(x))$$
$$-\frac{\kappa}{4\pi}\,\partial_i A_j(x)\varepsilon^{ij} - \partial_i P^{(1)i}(x) \approx 0 \tag{3.6c}$$

and β^a $(a = 1, 2, 3)$ are three arbitrary parameters.

The two second-class constraints read

$$\Omega_{(\alpha)}(x) = \Pi_{(\alpha)}(x) - i\left(\frac{a-1}{2}\right)\gamma_0\psi_{(\alpha)} \approx 0 \tag{3.7a}$$

$$\bar{\Omega}_{(\alpha)}(x) = \bar{\Pi}_{(\alpha)}(x) + i\left(\frac{a+1}{2}\right)\bar{\psi}_{(\alpha)}\gamma_0 \approx 0 \tag{3.7b}$$

4. DIAGRAMMATIC AND FEYNMAN RULES FROM PATH-INTEGRAL QUANTIZATION

Now we must carry out the path-integral quantization. This can be done by generalizing the Faddeev–Senjanovic method (Faddeev, 1970; Senjanovic, 1976) so that it is available for higher derivative field theories. The partition function we propose for higher derivative theories reads

$$Z = \int \mathscr{D}A_\mu^{(1)}\, \mathscr{D}P^{(1)\mu}\, \mathscr{D}A_\nu^{(2)}\, \mathscr{D}P^{(2)\nu}\, \mathscr{D}\bar{\psi}_{(\alpha)}\, \mathscr{D}\Pi^{(\alpha)} \mathscr{D}\psi_{(\beta)}\, \mathscr{D}\bar{\Pi}^{(\beta)} \delta(\Phi_1)\delta(\Phi_2)\delta(\Phi_3)$$

$$\times\, \delta(f_1)\delta(f_2)\delta(f_3)\, \det[\Phi_1, \Phi_2, \Phi_3, f_1, f_2, f_3]_D\, \delta(\Omega_{(\alpha)})\delta(\Omega_{(\beta)})\, \det[\Omega_{(\alpha)}, \Omega_{(\beta)}]$$

$$\times \exp i\left[\int d^3x\, (A_\mu^{(2)} P^{(1)\mu} + \dot{A}_\nu^{(2)} P^{(2)\nu} + \dot{\bar{\psi}}\Pi + \bar{\Pi}\dot{\psi}) - H_E\right] \tag{4.1}$$

The quantity $H_E = \int d^2x\, \mathscr{H}_E$ in equation (4.1) is the extended Hamiltonian, the generator of time evolutions, and it is defined in terms of the Hamiltonian density:

$$\mathscr{H}_E = \mathscr{H}_{\text{can}} + \delta\Phi_1 + \bar{\lambda}_{(\alpha)}\Omega^{(\alpha)} + \bar{\Omega}^{(\alpha)}\lambda_{(\alpha)} \tag{4.2}$$

where δ is the bosonic Lagrange multiplier and $\bar{\lambda}$ and λ are the fermionic Lagrange multipliers, corresponding to the three primary constraints (3.3c), (3.3e), and (3.3f).

The quantities f_1, f_2, and f_3 are gauge-fixing conditions. As shown in Greco *et al.* (1994), a convenient set of such conditions compatible with the equations of motion and satisfying $\det[f_a, \Phi_b]_D \neq 0$ for all first-class constraints Φ_a is

$$f_1 = \partial_i A^{(1)i} \approx 0 \tag{4.3a}$$

$$f_2 = A_0^{(2)} \approx 0 \tag{4.3b}$$

$$f_3 = \frac{\kappa}{2\pi}\nabla^2 A_0^{(1)} + e\varepsilon_{ik}\partial^k(\bar{\psi}\gamma^i\psi) + \Box\left(1 - \frac{c^2}{\pi}\Box\right)\partial_k A_i^{(1)}\varepsilon^{ik} \approx 0 \tag{4.3c}$$

The determinants appearing in equation (4.1) can be explicitly computed and also one can take path integral over the fields $A_0^{(2)}$, $P^{(1)\mu}$, $P^{(2)\mu}$, $\bar{\Pi}^{(\alpha)}$, and $\Pi^{(\alpha)}$, so one finds for the partition function

$$Z = \int \mathscr{D}A_\mu^{(1)}\, \mathscr{D}A_i^{(2)}\, \mathscr{D}\bar{\psi}_{(\alpha)}\, \mathscr{D}\psi_{(\beta)}\, \delta(f_1)\, \delta(f_3)\, \exp i\left[\int d^3x\, \mathscr{L}_{\text{eff}}\right] \tag{4.4}$$

The effective Lagrangian density $\mathscr{L}_{\rm eff}$ defined in (4.4) is given by

$$\begin{aligned}\mathscr{L}_{\rm eff} = & -\frac{1}{4}F_{ij}F^{ij} - \frac{c^2}{4\pi}G_{ij}G^{ij} - \frac{1}{2}(A_i^{(2)} - \partial_i A_0^{(1)})(A^{i(2)} - \partial^i A_0^{(1)}) - \frac{c^2}{2\pi}\dot{A}_i^{(2)}\dot{A}^{(2)i} \\ & -\frac{c^2}{4\pi}\partial_i F_{jk}\partial^i F^{jk} + \frac{c^2}{2\pi}A_j^{(2)}\nabla^2 A^{(2)j} - \frac{c^2}{\pi}\nabla^2 A_j^{(2)}\partial^j A_0^{(1)} - \frac{c^2}{2\pi}(\nabla^2 A_0^{(1)})^2 \\ & -\frac{\kappa}{4\pi}\partial_i A_0^{(1)}A_j^{(1)}\varepsilon^{ij} + \frac{\kappa}{4\pi}(\partial_i A_j^{(1)})A_0^{(1)}\varepsilon^{ij} + \frac{\kappa}{4\pi}A_i^{(2)}A_j^{(1)}\varepsilon^{ij} \\ & + i\left(\frac{a+1}{2}\right)\bar{\psi}\gamma^\mu\partial_\mu\psi + i\left(\frac{a-1}{2}\right)(\partial_\mu\bar{\psi})\gamma^\mu\psi - m\bar{\psi}\psi + e\bar{\psi}\gamma_\mu\psi A^{(1)\mu} \end{aligned} \tag{4.5}$$

where $G_{ij} = \partial_i A_j^{(2)} - \partial_j A_i^{(2)}$ and $\nabla^2 = \partial_i\partial^i$.

By looking at equation (4.4), it can be seen that the quantum problem is defined in terms of a path integral in which there are four independent fields. Consequently, it is possible to apply diagrammatic techniques defining proper Feynman rules for propagators and vertices corresponding to the fields $A_\mu^{(1)}$, $A_i^{(2)}$, $\bar{\psi}_{(\alpha)}$, and $\psi_{(\alpha)}$.

Alternatively, the path integral equation (4.4) can be written

$$Z = \int \mathscr{D}A_\mu^{(1)}\,\mathscr{D}A_i^{(2)}\,\mathscr{D}\bar{\psi}_{(\alpha)}\,\mathscr{D}\psi_{(\beta)}\,\mathscr{D}\Lambda_1\,\mathscr{D}\Lambda_3 \exp i\left[\int d^3x\;\mathscr{L}^*\right] \tag{4.6}$$

where

$$\mathscr{L}^* = \mathscr{L}_{\rm eff} - \Lambda_1 f_1 - \Lambda_3 f_3 \tag{4.7}$$

for the Lagrange multipliers Λ_1 and Λ_3.

As carried out in a different context (Dobry *et al.*, 1990; Greco and Zandron, 1991), we can define a bosonic vector quantity X_Σ, having the same dimension as the vector field $A_\mu^{(1)}$, whose components are given by

$$X_\Sigma = \left(A_\mu^{(1)}, cA_i^{(2)}, c\Lambda_1, \frac{1}{c}\Lambda_3\right) \tag{4.8}$$

where the compound index Σ takes the seven values $0, 1, \ldots, 6$.

Therefore, when the action is written in terms of the vector quantity (4.8) we can easily recognize the propagators defined by the quadratic part of the Lagrangian (4.7) and the remaining part can be represented by vertices ('t Hooft and Velman, 1973). Consequently, equation (4.7) can be seen as the Lagrangian density which defines the effective action for a system describing the boson vector field X_Σ coupled to a matter Dirac spinor field. The effective action $\mathscr{S}^*$ can be written

$$\mathscr{S}^* = \mathscr{S}^*(X_\Sigma) + \mathscr{S}^*(\psi) + \mathscr{S}^*_{\rm int}(X_\Sigma, \psi) \tag{4.9}$$

where, taking into account equation (4.5), one has

$$\mathcal{S}^*(\psi) = \int d^3x\ \mathcal{L}_f \tag{4.10a}$$

$$\mathcal{S}^*(X_\Sigma, \psi) = \int d^3x\ [e\bar{\psi}(\Gamma_\Sigma X^\Sigma)\psi] \tag{4.10b}$$

$$\mathcal{S}^*(X_\Sigma) = \int d^3x \left[\frac{1}{2} X_\Sigma (D^{-1})^{\Sigma\Lambda} X_\Lambda\right] \tag{4.10c}$$

The seven matrices $\Gamma_\Sigma = (\Gamma_{A_0}, \Gamma_{A_i}, \Gamma_{B_i}, \Gamma_{\Lambda_1}, \Gamma_{\Lambda_3})$ defined in equation (4.10b) are

$$\Gamma_{A_0} = \gamma_0, \qquad \Gamma_{A_i} = \gamma_i, \qquad \Gamma_{B_i} = 0, \qquad \Gamma_{\Lambda_1} = 0, \qquad \Gamma_{\Lambda_3} = c\gamma^i \varepsilon_{ik} \partial^k \tag{4.11}$$

The 7×7 matrix $(D^{-1})^{\Sigma\Lambda}$ defined in equation (4.10c) is the inverse of the propagator of the bosonic object X_Σ given in (3.15). It is Hermitian, nondegenerate, and invertible and so the propagator $D_{\Sigma\Lambda}(k)$, in the momentum space, can be evaluated. The computation of the matrix elements $D_{\Sigma\Lambda}(k)$ of the propagator in the general case is straightforward but very tedious. These were obtained by using REDUCE 3.2 and are long algebraic expressions which we do not write here explicitly.

In the simpler $\kappa = 0$ case, $\det[D^{-1}(k)]$ in the momentum space is given by

$$\det[D^{-1}(k)] = c^2 k_0^2 k^8 (1 - c^2 k^2)^2 \varepsilon \left(\varepsilon - \frac{1}{c^2}\right)\left[\varepsilon - \left(\frac{1}{c^2} - k^2\right)\right] \tag{4.12}$$

where $\varepsilon = k_1^2 + k_2^2 - k_0^2 = k^2 - k_0^2$. From now on we rename the parameter c in such a way that it is not necessary to write the constant π explicitly.

For $\kappa = 0$, we have computed the following matrix elements of the boson propagator $D_{\Sigma\Lambda}(k)$:

$$D_{00} = \frac{1 - c^2\varepsilon}{(1 - c^2k^2)c^2k^2k_0^2}, \qquad D_{03} = -D_{30} = \frac{ik_1}{ck^2k_0^2}, \qquad D_{04} = -D_{40} = \frac{ik_2}{ck^2k_0^2}$$

$$D_{11} = \frac{k_0^2k_2^2(1 - c^2k^2 - c^2\varepsilon)}{k^4\varepsilon(1 - c^2\varepsilon)(1 - c^2k^2)}, \qquad D_{12} = D_{21} = -\frac{k_0^2k_1k_2(1 - c^2k^2 - c^2\varepsilon)}{k^4\varepsilon(1 - c^2\varepsilon)(1 - c^2k^2)}$$

$$D_{13} = -D_{31} = \frac{ick_0k_2^2}{k^2\varepsilon(1 - c^2\varepsilon)}, \qquad D_{14} = -D_{41} = \frac{-ick_0k_1k_2}{k^2\varepsilon(1 - c^2\varepsilon)}$$

$$D_{15} = -D_{51} = \frac{ick_1}{k^2}, \qquad D_{16} = -D_{61} = \frac{-ik_2}{ck^2\varepsilon(1 - c^2\varepsilon)}$$

$$D_{22} = \frac{k_0^2k_1^2(1 - c^2k^2 - c^2\varepsilon)}{k^4\varepsilon(1 - c^2\varepsilon)(1 - c^2k^2)}, \qquad D_{23} = -D_{32} = \frac{-ick_0k_1k_2}{k^2\varepsilon(1 - c^2\varepsilon)}$$

$$D_{24} = -D_{42} = \frac{ick_0 k_1^2}{k^2 \varepsilon(1 - c^2\varepsilon)}, \qquad D_{25} = -D_{52} = \frac{ick_2}{k^2}$$

$$D_{26} = -D_{62} = \frac{ik_1}{ck^2 \varepsilon(1 - c^2\varepsilon)}, \qquad D_{36} = D_{63} = \frac{-k_0 k_2}{k^2 \varepsilon(1 - c^2\varepsilon)}$$

$$D_{33} = \frac{1}{k_0^2}\left[1 - \frac{k_2^2}{k^2} + \frac{c^2 k_2^2 k_0^2 (1 - c^2 k^2)}{\varepsilon(1 - c^2\varepsilon)(1 - c^2 k^2 - c^2\varepsilon)}\right]$$

$$D_{34} = D_{43} = \frac{k_1 k_2}{k_0^2 k^2} - \frac{k_1 k_2 c^2 (1 - c^2 k^2)}{\varepsilon(1 - c^2\varepsilon)(1 - c^2 k^2 - c^2\varepsilon)}$$

$$D_{44} = \frac{1}{k_0^2}\left[1 - \frac{k_1^2}{k^2} + \frac{c^2 k_1^2 k_0^2 (1 - c^2 k^2)}{\varepsilon(1 - c^2\varepsilon)(1 - c^2 k^2 - c^2\varepsilon)}\right]$$

$$D_{46} = D_{64} = \frac{k_0 k_1}{k^2 \varepsilon(1 - c^2\varepsilon)}, \qquad D_{66} = \frac{1}{c^2 k^2 \varepsilon(1 - c^2\varepsilon)} \qquad (4.13)$$

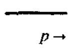
and all the others vanish.

We can write the Feynman rule propagators and vertices.

(i) Propagators: We associate with the propagator $D_{\Sigma\Lambda}$ of the bosonic field X_Σ a wavy line connecting two generic points:

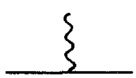
X_Σ $X_\Lambda \equiv D_{\Sigma\Lambda}(k)$

and with a straight line the usual propagator of the fermionic field ψ,



$$\equiv \frac{-i}{\gamma \cdot p + m} = \frac{i(\gamma \cdot p - m)}{p^2 + m^2}$$

(ii) Vertices: Then, the three-leg vertex of the model is

 $\equiv -ie\Gamma_\Sigma$

Moreover, as usual we have to take into account a minus sign for every closed fermion loop and another minus sign for diagrams related to the exchange of two fermion lines, internal or external. A combinatorial factor correcting for double counting in case identical particles occur also must be taken into account.

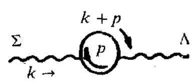
5. PERTURBATIVE METHOD IN HIGHER DERIVATIVE QED. ONE-LOOP STRUCTURE

Now we examine the perturbative treatment of this gauge theory which describes the interaction of the bosonic object X_Σ with the Dirac spinor field. Using the above rules, a power-counting analysis shows that the superficial degrees of divergence G are essentially those of the QED, so we are led to the following one-loop diagrams:

which correct the fundamental parameters and fields of the theory.

If we call $\Pi_{\Sigma\Lambda}(k)$ the correction to the boson line or vacuum polarization diagram, we can write

$\Pi_{\Sigma\Lambda}(k) =$

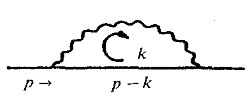


$$= -e^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{(p^2+m^2)[(p+k)^2+m^2]}$$
$$\times \mathrm{Tr}[\Gamma_\Sigma(\gamma \cdot p + \gamma \cdot k - m)\Gamma_\Lambda(\gamma \cdot p - m)] \qquad (5.1)$$

By looking at equations (4.11) for the Γ_Σ matrices, we conclude that the integral expression for the correction to the boson line is similar to the model without higher derivatives; therefore the ultraviolet behavior of the integral (5.1) is the same as in usual QED, i.e., for large momentum p, the Feynman integral (5.1) behaves as $\sim \int dp$ and so this diagram is linearly divergent. This must be expected, because this model is higher derivative only with respect to the boson field, and the propagator of that field does not appear in the vacuum polarization diagram. Consequently, the evaluation of the integral (5.1) is carried out by introducing a Feynman parameter and the new loop momentum $p' = p + kx$. We note that in the case in which the complete CS Lagrangian (3.2a) (with $\kappa \neq 0$) is considered, the dimensional regularization cannot be used safely due to the presence of the volume form $\varepsilon^{\mu\nu\rho}$. In that case, another gauge-invariant regularization method, for instance the Pauli–Villars one (Alvarez-Gaumé *et al.*, 1990), must be used. The renormalization procedure is implemented as in usual QED.

Now we consider the second diagram given above. Let $\Sigma(p)$ be the correction to the fermion line (suppressing spinor indices). This diagram is given by the following integral expression:

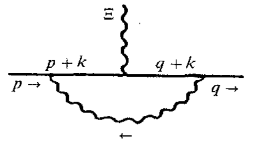
$\Sigma(p) =$



$$= -ie^2 \int \frac{d^3k}{(2\pi)^3} \frac{\Gamma_\Sigma(\gamma \cdot p - \gamma \cdot k - m)\Gamma_\Lambda}{(p-k)^2+m^2} \times D_{\Sigma\Lambda}(k) \qquad (5.2)$$

When the explicit sum on the Greek indices Σ and Λ is carried out, we can see that we have only the propagators $D_{00}(k)$, $D_{11}(k)$, $D_{12}(k)$, $D_{22}(k)$, and $D_{66}(k)$. From equations (4.13), it can be seen that, for large momentum, the propagators D_{00}, D_{11}, D_{12}, and D_{22} behave like k^{-4}; while the propagator D_{66} behaves like k^{-6}. Therefore the new propagators have an ultraviolet behavior in such a manner that the Feynman integral (5.2) gives a convergent result (for large k the integral behaves as $\sim \int dk/k^3$).

The last one-loop diagram we consider is the vertex correction, which we call $V_\Xi(p, q)$:

$V_\Xi(p, q) =$ 

$$= -ie^3 \int \frac{d^3k}{(2\pi)^3} \Gamma_\Sigma \frac{(\gamma \cdot p + \gamma \cdot k - m)}{(p+k)^2 + m^2} \Gamma_\Xi \frac{(\gamma \cdot q + \gamma \cdot k - m)}{(q+k)^2 + m^2} \Gamma_\Lambda \times D_{\Sigma\Lambda}(k) \tag{5.3}$$

Once more, by introducing the expressions (4.13) for the boson propagators $D_{\Sigma\Lambda}$ and the Γ matrices (4.11) and carrying out the summation over the indices, we can see that the diagram is also convergent, because for large momentum the integral behaves as $\sim \int dk/k^4$.

We conclude that in the one-loop diagrams in which the propagator $D_{\Sigma\Lambda}$ of the boson X_Σ occurs, the ultraviolet behavior is improved and the divergence of these diagrams is eliminated. The remaining diagrammatic with a fermionic loop does not change and the degree of divergence is that of usual QED. Therefore, the use of higher derivative terms in the Lagrangian allows us to improve the behavior of the correspondent propagators at large momentum, rendering the theory less divergent.

6. UNITARITY

At this stage, another important problem to take into account is the unitarity problem. It is well known (Hawking, 1987) that in higher derivative theories, the unitarity can be violated when ghost states with negative norm are present. To say something about the unitarity, we must analyze more carefully the bosonic propagator we have defined.

First we note that the Hermitian matrix $D_{\Lambda\Sigma}(k)$ whose elements were defined in (4.11) can be diagonalized and so its eigenvalues are determined. The corresponding secular equation depends only on k^2 and k_0 and therefore we can study the problem for a given value of the three-vector k_μ. Let $k_\mu = (k_0, k_1 = 0, k_2)$ be the value for the three-vector momentum. The resulting matrix can be "diagonalized" in three blocks: two 2×2 blocks and one 3×3 block. The matrix elements which correspond to the two 2×2 blocks do not contain any single pole. Therefore, such elements do not have corresponding S-matrix elements. The 3×3 block has the three single poles which appear in the theory. If we write

$$\Delta = c^4 k^2 \varepsilon \left(\varepsilon - \frac{1}{c^2} \right) \left[\varepsilon - \left(\frac{1}{c^2} - k^2 \right) \right] \tag{6.1}$$

then we can write the 3×3 matrix

$$K_{ab} = \frac{1}{\Delta} \times K^R_{ab} \tag{6.2}$$

The matrix residue K^R_{ab} $(a, b = 1, 2, 3)$ is given by

$$\begin{bmatrix} \frac{k_0^2}{1 - c^2 k^2}(1 - c^2k^2 - c^2\varepsilon)^2 & -i\frac{k}{c}(1 - c^2k^2 - c^2\varepsilon) & ik_0 k^2 c(1 - c^2k^2 - c^2\varepsilon) \\ i\frac{k}{c}(1 - c^2k^2 - c^2\varepsilon) & \frac{1}{c^2}(1 - c^2k^2 - c^2\varepsilon) & -k_0 k(1 - c^2k^2 - c^2\varepsilon) \\ -ik_0 k^2 c(1 - c^2k^2 - c^2\varepsilon) & -k_0 k(1 - c^2k^2 - c^2\varepsilon) & c^2k^4(1 - c^2k^2) \end{bmatrix}$$

The matrix $K_{ab}(k)$ has three single poles in the values of ε: $\varepsilon = 0$, $\varepsilon = 1/c^2$ and $\varepsilon = 1/c^2 - k^2$, as can be seen from (6.1).

The matrix residue $K^R_{ab}(k)$ can be diagonalized and has three different nonzero eigenvalues $\xi_{(\alpha)}$ $[(\alpha) = 1, 2, 3]$. Consequently, we can define a set of real currents $J^{(\alpha)}_a(k)$, one for every nonzero eigenvalue, which are mutually orthogonal and eigenstates of the matrix $K^R_{ab}(k)$ ('t Hooft and Velman, 1973), i.e.,

$$J^{(\alpha)}_a(k) J^{(\beta)}_a(k) = 0 \qquad \text{if} \quad (\alpha) \neq (\beta) \tag{6.3a}$$

$$K^R_{ab}(k) J^{(\alpha)}_b(k) = e^{(\alpha)}(k) J^{(\alpha)}_a(k) \tag{6.3b}$$

For instance, the real bosonic currents for the emission of a particle corresponding to incoming particles of the S-matrix, when all the eigenvalues of the matrix K^R are positive, must be normalized in such a way that

$$J^{(\alpha)}_a(k) K^R_{ab}(k) J^{(\alpha)}_b(k) = +1 \tag{6.4}$$

The source currents thus defined are properly normalized for emission of a particle (or an antiparticle). Of course, when the absorption of a particle (or an antiparticle) is considered, in the matrix K^R appearing in equations (6.3) the momentum k must be replaced by $-k$.

On the other hand, once the matrix residue K^R_{ab} is diagonalized, the above equations (6.3) imply that the currents are of the form:

$$J^{(\alpha)} = (0, 1/(\xi_{(\alpha)})^{1/2}, 0)$$

with the obvious result

$$\sum_{(\alpha)} J^{(\alpha)} J^{(\alpha)} = [K^R]^{-1} \tag{6.5}$$

Equation (6.5) holds when the eigenvalues of the matrix K^R at the pole are positive, that is, when unitarity is preserved and the normalization (6.4) holds. In the case of negative eigenvalues, to recover unitarity, the normalization in (6.4) must be done with a minus one. As is well known, when the matrix residue K^R has a negative eigenvalue at the pole, it corresponds to states with negative norm and they are physically unacceptable. The corresponding particles are called ghosts. This is the prescription to retrieve the unitarity of the theory and it is usually called the indefinite metric prescription.

Let us consider the secular equation corresponding to the matrix K^R,

$$\begin{aligned}
&\xi^3 - \frac{1}{(1-c^2k^2)c^2}\,\xi^2[1 - c^2k^2 - c^6k^6 + c^8k^8 \\
&\quad - c^2\varepsilon(2 - c^2k^2 - c^4k^4) + c^4\varepsilon^2(2 - c^2k^2) - c^6\varepsilon^3] \\
&\quad - \frac{1}{(1-c^2k^2)c^2}\,\xi\varepsilon(1 - c^2\varepsilon)(1 - c^2k^2 - c^2\varepsilon)(1 - c^4k^4 - c^2\varepsilon) \\
&\quad + \frac{k^2}{1-c^2k^2}\,\varepsilon^2(1 - c^2\varepsilon)^2(1 - c^2k^2 - c^2\varepsilon)^2 = 0
\end{aligned} \tag{6.6}$$

The three eigenvalues can be given, for example, as power series of ε whose coefficients are functions of k^2. From equation (6.6) it is possible to analyze the residue of the eigenvalues at every pole. For instance, if we work with the dimensional parameter c in regimes satisfying the condition $c^2k^2 < 1$, we can assert that the residue at the pole $\varepsilon = 0$ is

$$\frac{1}{c^2}(1 - c^6k^6) > 0 \tag{6.7a}$$

Therefore, in this case the normalization in (6.4) must be given with plus one.

Similarly the residues at the poles $\varepsilon = 1/c^2$ and $\varepsilon = (1/c^2 - k^2)$ are, respectively,

$$-k^2(1 + c^4k^4) < 0 \tag{6.7b}$$

and

$$c^2k^4(1 - c^2k^2) > 0 \tag{6.7c}$$

Finally, we note that it is easy to show that the set of constraints is modified when the limit $c = 0$ is taken in the system. The fermionic constraints do not change, but the bosonic constraints (3.6a) disappear in this case, with $P^{(1)0} = 0$ as the unique primary bosonic constraint. The consistency condition on the constraints gives only one secondary constraint. It is possible to find the two first-class constraints characteristic of usual electrodynamics with the CS term. The two corresponding gauge-fixing conditions are given by $f_1 = \partial_i A^{(1)i} \approx 0$ and $f_2 = A_0^{(1)} \approx 0$. The partition function analogous to (4.1), after integrating in the variable $A_0^{(1)}$ by using the function $\delta(f_2)$, reduces to

$$Z = \int \mathscr{D}\mathscr{A}_i^{(1)}\, \mathscr{D}\bar{\psi}\, \mathscr{D}\psi\, \delta(f_2) \exp i[S_{\text{eff}}] \tag{6.8}$$

where S_{eff} is now the effective action for electrodynamics with topological CS term.

If we look at the limit $c \to 0$ for the propagator (6.2), we can easily see that two of the singularities go to infinity and the only remaining pole is $\varepsilon = 0$, as expected.

7. CONCLUSIONS

Continuing the work started in Greco *et al.* (1994), in the present paper we found the Feynman rules and the diagrammatic for a higher-derivative Chern–Simons theory coupled to matter. This was done in the framework of the path integral quantization method. The definition of the effective Lagrangian density allows us to define a suitable "bosonic field" and to find the propagator of such a bosonic object. The fermionic propagator for the matter field is the usual one. The model has a unique three-leg vertex and so all the diagrams are obtained by connecting vertices and sources by means of the propagators thus defined. Using the perturbative theory, we analyzed the one-loop structure of the model. The results obtained for the one-loop diagrams in which the boson field propagator occurs allow us to guarantee that the ultraviolet behavior is improved and the divergence of these diagrams is eliminated. Therefore, we conclude that the presence of higher derivative terms in the Lagrangian density gives rise to a new bosonic propagator, which makes the theory less divergent.

We have also shown, unlike what occurs in usual quantum electrodynamics, that in this higher derivative model there are three single poles. Two of these poles go to infinity in the limit $c \to 0$, recovering the electrodynamic singularity.

Finally, we also gave a prescription to eliminate the ghost states with negative norm in such a way that unitarity can be preserved.

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